2026

Four Year Under Graduate Programme (FYUGP)

2nd Semester Examination (Under NEP)
(New Session 2024-25)

MATHEMATICS (Major)

Paper Code : MTMMJ DC-201

(Calculus and Analytical Geometry)

Full Marks : 50 — Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2\times5=10$

(a) Does $\lim\limits_{x\to0}\dfrac{1}{1+e^{\frac{1}{x}}}$ exist? Justify your answer.

(b) Find the n-th derivative of $y=\dfrac{1}{x^2-a^2}$.

(c) Show that $\int_0^{\frac{\pi}{2}}\sin^5 x\,dx=\dfrac{8}{15}$.

P.T.O.

(d) Prove that $\beta\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$.

(e) For what values of λ, the equation

$$x^2 + \lambda xy - 2y^2 + 3y - 1 = 0$$

represents a pair of straight lines?

(f) Find the common tangents to the hyperbolas

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

(g) Find the equation of the sphere passing through the points (0,0,0), (a,0,0), (0,b,0), (0,0,c).

(h) Find the equation of the right circular cylinder whose axis is $\frac{x}{1} = \frac{y}{-2} = \frac{z}{2}$ and radius equal to 2.

2. (a) If $y = a\cos(\log x) + b\sin(\log x)$, prove that

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0.$$

(b) If $I_{m,n} = \int_0^1 x^m (1-x)^n \, dx$, where $m, n \in \mathbb{N}$, prove that $(m+n+1)I_{m,n} = nI_{m,n-1}$ and hence deduce that $I_{m,n} = \frac{m!n!}{(m+n+1)!}$. 5+5

3. (a) Find the range of values of x for which the curve

$$y = x^4 - 16x^3 + 42x^2 + 12x + 1$$

is concave or convex with respect to the x-axis and identify points of inflexion, if any.

(b) Find the length of the perimeter of the cardioid $r = a(1-\cos\theta)$. Show that the upper half of the curve is bisected at $\theta = \frac{2\pi}{3}$. 5+(3+2)

4. (a) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines, show that the area of the triangle formed by the bisectors of the angle between them and the axis of x is

$$\frac{\sqrt{(a-b)^2 + 4h^2}}{2h} \cdot \frac{ac - g^2}{ab - h^2}.$$

(b) If any tangent plane to the sphere

$$x^2 + y^2 + z^2 = r^2$$

makes intercepts a, b and c on the coordinate axes, then prove that $\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{1}{r^2}$. 5+5

5. (a) Prove that

$$\Gamma\left(\tfrac{1}{2}\right)\Gamma(2x) = 2^{2x-1}\Gamma(x)\Gamma\left(x + \frac{1}{2}\right);\ x > 0.$$

(b) Find the asymptotes of the curve

$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0.$$

5+5

6. (a) The section of the cone whose guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ by the plane $x=0$ is a rectangular hyperbola. Show that the locus of the vertex is the surface $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1$.

(b) An astroid $x=a\cos^3\theta, y=a\sin^3\theta$ from $\theta=0$ to $\theta=\frac{\pi}{2}$ revolves about the x-axis; show that the volume of the solid generated is $\frac{16\pi a^3}{105}$.

(c) Find the vertical asymptotes of the curve $y=xe^{\frac{1}{x}}$.

5+4+1

7. (a) Find the envelope of the family of the ellipses $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where the parameters a, b are connected by $a^m+b^m=c^m$, m is a positive integer.

(b) Show that the straight line $r\cos(\theta-\alpha)=p$ touches the conic $\frac{l}{r}=1+e\cos\theta$ if

$$(l\cos\alpha-pe)^2+l^2\sin^2\alpha=p^2.$$

5+5